統計学概論　解答例

2011.10.11

問題　次を示せ．

(1) $m\,{}_nC_m = n\,{}_{n-1}C_{m-1} \qquad (1 \leq m \leq n)$

(2) ${}_nC_m = {}_{n-1}C_{m-1} + {}_{n-1}C_m \qquad (1 \leq m < n)$

(3) ${}_{n+1}C_{m+1} = {}_nC_m + {}_{n-1}C_m + \cdots + {}_{m+1}C_m + {}_mC_m \qquad (0 \leq m \leq n)$

(4) $\displaystyle\sum_{k=0}^{n} {}_nC_k = 2^n \qquad (n \geq 1)$

(5) $\displaystyle\sum_{k=0}^{n} (-1)^k\ {}_nC_k = 0 \qquad (n \geq 1)$

（解）二項係数 ${}_nC_k$ の定義より

$$\begin{aligned}
m\,{}_nC_m =& \frac{m \cdot n!}{m!\,(n-m)!} = \frac{n \cdot (n-1)!}{(m-1)!\,\{(n-1)-(m-1)\}!} = n\,{}_{n-1}C_{m-1} \qquad (1 \leq m \leq n)\,, \\
{}_nC_m =& \frac{\{m+(n-m)\} \cdot (n-1)!}{m!\,(n-m)!} = \frac{(n-1)!}{(m-1)!\,\{(n-1)-(m-1)\}!} + \frac{(n-1)!}{m!\,\{(n-1)-m\}!} \\
=& {}_{n-1}C_{m-1} + {}_{n-1}C_m \qquad (1 \leq m < n)
\end{aligned}$$

が得られ，(1) および (2) が成り立つ．(2) を帰納的に適用し，${}_{m+1}C_{m+1} = 1 = {}_mC_m$ を用いると，

$$\begin{aligned}
{}_{n+1}C_{m+1} =& {}_nC_m + {}_nC_{m+1} = {}_nC_m + {}_{n-1}C_m + {}_{n-1}C_{m+1} = {}_nC_m + {}_{n-1}C_m + {}_{n-2}C_m + {}_{n-2}C_{m+1} \\
=& \cdots = {}_nC_m + {}_{n-1}C_m + {}_{n-2}C_m + \cdots + {}_{m+1}C_m + {}_{m+1}C_{m+1} \\
=& {}_nC_m + {}_{n-1}C_m + {}_{n-2}C_m + \cdots + {}_{m+1}C_m + {}_mC_m
\end{aligned}$$

となり，(3) が成り立つ．二項定理

$$(a+b)^n = \sum_{k=0}^{n} {}_nC_k\, a^k\, b^{n-k}$$

より

$$\begin{aligned}
2^n =& (1+1)^n = \sum_{k=0}^{n} {}_nC_k \cdot 1^k \cdot 1^{n-k} = \sum_{k=0}^{n} {}_nC_k, \\
0 =& \{(-1)+1\}^n = \sum_{k=0}^{n} {}_nC_k \cdot (-1)^k \cdot 1^{n-k} = \sum_{k=0}^{n} (-1)^k\ {}_nC_k
\end{aligned}$$

となり，(4) および (5) が成り立つ．■