

確率統計学1 課題 解答例

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1 $n \geq 2$ に対して次を求めよ.

$$(1) \sum_{k=0}^n {}_n C_k \quad (2) \sum_{k=0}^n k {}_n C_k \quad (3) \sum_{k=0}^n k(k-1) {}_n C_k \quad (4) \sum_{k=0}^n k^2 {}_n C_k$$

(解) (1) 二項定理より

$$\sum_{k=0}^n {}_n C_k = \sum_{k=0}^n {}_n C_k 1^k 1^{n-k} = (1+1)^n = 2^n$$

となる. (2) 変数変換 $\ell = k - 1$ により

$$\begin{aligned} \sum_{k=0}^n k {}_n C_k &= \sum_{k=1}^n k {}_n C_k = \sum_{\ell=0}^{n-1} (\ell+1) {}_n C_{\ell+1} = \sum_{\ell=0}^{n-1} \frac{(\ell+1)n!}{(\ell+1)!(n-\ell-1)!} \\ &= \sum_{\ell=0}^{n-1} \frac{n \cdot (n-1)!}{\ell! (n-\ell-1)!} = n \sum_{\ell=0}^{n-1} {}_{n-1} C_{\ell} = n 2^{n-1} \end{aligned}$$

が得られる. (3) 変数変換 $\ell = k - 2$ により

$$\begin{aligned} \sum_{k=0}^n k(k-1) {}_n C_k &= \sum_{k=2}^n k(k-1) {}_n C_k = \sum_{\ell=0}^{n-2} (\ell+2)(\ell+1) {}_n C_{\ell+2} = \sum_{\ell=0}^{n-2} \frac{(\ell+2)(\ell+1)n!}{(\ell+2)!(n-\ell-2)!} \\ &= \sum_{\ell=0}^{n-1} \frac{\{n(n-1)\} \cdot (n-2)!}{\ell! (n-\ell-2)!} = n(n-1) \sum_{\ell=0}^{n-2} {}_{n-2} C_{\ell} = n(n-1) 2^{n-2} \end{aligned}$$

が得られる. (4) 問 (2), (3) より

$$\sum_{k=0}^n k^2 {}_n C_k = \sum_{k=0}^n k(k-1) {}_n C_k + \sum_{k=0}^n k {}_n C_k = n(n-1) 2^{n-2} + n 2^{n-1} = n(n+1) 2^{n-2}$$

となる. ■