解析学１　解答例

2018.11.13

■　関数 $f(x,y)$ の 2 回までのすべての偏導関数が存在し，それらは連続であるとする．このとき，$F(\theta)=f(\cos\theta,\sin\theta)$ の導関数 $\dfrac{dF}{d\theta}(\theta)$ および 2 階導関数 $\dfrac{d^2F}{d\theta^2}(\theta)$ を求めよ．

（解）　合成関数の微分法により

$$
\begin{aligned}
\frac{dF}{d\theta}(\theta) =& \frac{\partial f}{\partial x}(\cos\theta,\sin\theta)\,(-\sin\theta) + \frac{\partial f}{\partial y}(\cos\theta,\sin\theta)\,\cos\theta \\
=& -\frac{\partial f}{\partial x}(\cos\theta,\sin\theta)\,\sin\theta + \frac{\partial f}{\partial y}(\cos\theta,\sin\theta)\,\cos\theta, \\
\frac{d^2F}{d\theta^2}(\theta) =& -\left\{\frac{\partial^2 f}{\partial x^2}(\cos\theta,\sin\theta)\,(-\sin\theta) + \frac{\partial^2 f}{\partial x\partial y}(\cos\theta,\sin\theta)\,\cos\theta\right\}\sin\theta \\
& \quad + \left\{\frac{\partial^2 f}{\partial x\partial y}(\cos\theta,\sin\theta)\,(-\sin\theta) + \frac{\partial^2 f}{\partial y^2}(\cos\theta,\sin\theta)\,\cos\theta\right\}\cos\theta \\
& \quad - \frac{\partial f}{\partial x}(\cos\theta,\sin\theta)\,\cos\theta + \frac{\partial f}{\partial y}(\cos\theta,\sin\theta)\,(-\sin\theta) \\
=& \frac{\partial^2 f}{\partial x^2}(\cos\theta,\sin\theta)\,\sin^2\theta - 2\,\frac{\partial^2 f}{\partial x\partial y}(\cos\theta,\sin\theta)\,\sin\theta\,\cos\theta \\
& \quad + \frac{\partial^2 f}{\partial y^2}(\cos\theta,\sin\theta)\,\cos^2\theta - \frac{\partial f}{\partial x}(\cos\theta,\sin\theta)\,\cos\theta - \frac{\partial f}{\partial y}(\cos\theta,\sin\theta)\,\sin\theta
\end{aligned}
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である．■